

# Characteristics of Polyphonic Music Style and Markov Model of Pitch-Class Intervals

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**Abstract.** For the purpose of quantitatively characterising polyphonic music styles, we study computational analysis of some traditionally recognised harmonic and melodic features and their statistics. While a direct computational analysis is not easy due to the need for chord and key analysis, a method for statistical analysis is developed based on relations between these features and successions of pitch-class (pc) intervals extracted from polyphonic music data. With these relations, we can explain some patterns seen in the model parameters obtained from classical pieces and reduce a significant number of model parameters (110 to five) without heavy deterioration of accuracies of discriminating composers in and around the common practice period, showing the significance of the features. The method can be applied for polyphonic music style analyses for both typed score data and performed MIDI data, and can possibly improve the state-of-the-art music style classification algorithms.

**Keywords:** polyphonic music analysis; pitch class interval; statistical music model; music style recognition; composer discrimination

## 1 Introduction

Harmonic and melodic features of polyphonic music have long been recognised to characterise music styles in and around the common practical period [1–3]. A quantitative and computational method of analysing these features would yield applications such as music style/genre recognition. However, a direct analysis is not easy because it requires chord and key recognition techniques, which are still topics of developing research [4, 5]. Music style/genre classification has recently been gathering attentions in music information processing (e.g. [6–11]), but there is still much room for researches in incorporating/relating and traditional knowledge in music theory and musicology to computational models. How to extract effective features from a generic polyphonic music data including performed MIDI data with temporal fluctuations of notes is also an open problem. In this study, we relate four traditionally recognised features of polyphonic music to computationally extractable elements of generic polyphonic MIDI data and develop a method for statistical analysis based on these elements.

## 2 Polyphonic Features and Successions of PC Intervals

We list four commonly studied features of polyphonic tonal music regarding harmony and melody [1–3], which will be called polyphonic features:

- F1 Dissonant chords and motions:** Use of dissonant chords and motions is generally more severely constrained in older music.
- F2 Non-diatonic motions:** These include successive semi-tone-wise motions and a succession of major third, etc. and characterise music styles.
- F3 Modulations:** The type and frequency of modulations characterise composers and periods.
- F4 Non-harmonic notes:** Their usage and frequency characterise composers and periods.

In order to study these features efficiently for generic music data, we only consider intervals of the pitch classes (pcs) and disregard other elements including durations as the subject of analysis. We assume that the data is represented as a sequence of integral pitches (with the identification of enharmonic equivalents) ordered according to their onset times. If there are several notes with simultaneous onset times, we prescribe that they can be ordered in any way. Any data, either typed scores or recorded performances, given in MIDI format can be used. The sequence of pc intervals is obtained by applying the modulo operation of divisor 12 and then taking intervals. Because data points with a zero pc interval express little about the polyphonic features, they are dropped and we have a reduced sequence of pc intervals denoted by  $\mathbf{x} = (x_n)_{n=1}^N$  ( $x_n = 1, \dots, 11$ ).

A dissonant interval in a chord can be expressed by a pc interval  $x = 1, 2, 6, 10, 11$  within the chord. Since only tritone is a definite dissonant interval in a melodic motion with the identification of enharmonic equivalents,  $x = 6$  is the only direct indication of a dissonant motion. Based on these facts, the distribution of pc intervals is used to characterise music styles in Ref. [6].

Extending this basic result, more abundant information on the polyphonic features can be extracted from the successions of pc intervals (hereafter PCI successions). For example, a chord containing G, B, and F could be represented by a succession (F,G,B), and correspondingly, (2, 4) in the sequence of pc intervals. The tritone is implicit as a pc interval but appears indirectly as a composite interval of  $2+4 = 6$ . A similar case appears in an indirect melodic motion involving a tritone and in false relations involving a tritone. These cases be generalised to a succession of two pc intervals  $(x_n, x_{n+1})$  with  $x_n + x_{n+1} = 1, 2, 6, 10, 11$ .

A diatonic motion of pitches can be defined as a sequence of pitches which can be embedded in a diatonic (or major) scale, and a non-diatonic motion is defined conversely. Any pc interval can result from a diatonic motion: A pc interval 1 can correspond to  $m2^1$  (we express this as  $1 \rightarrow m2$ ), and similarly,  $2 \rightarrow M2$ ,  $3 \rightarrow m3$ ,  $4 \rightarrow M3$ ,  $5 \rightarrow P4$ ,  $6 \rightarrow a4, d5$ ,  $7 \rightarrow P5$ ,  $8 \rightarrow m6$ ,  $9 \rightarrow M6$ ,  $10 \rightarrow m7$ ,  $11 \rightarrow M7$ . By contrast, certain successions appear only in non-diatonic

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<sup>1</sup> We use abbreviations for diatonic intervals such as m2 is minor second, M2 is major second, P4 is perfect fourth, a4 is augmented fourth, d5 is diminished fifth, etc.

**Table 1.** Classes of successions of two pc intervals and their relation with the polyphonic features.

Label	Name	Member	Polyphonic feature	Related class(es)
C1	Succession to tritone/ semitone/whole tone	$\{(x, y)   y=6, 1, 11, 2, 10\}$		
C2	Indirect octave	$\{(x, y)   x+y=0 \pmod{12}\}$	F1	C1, C3
C3	Indirect tritone/ semitone/whole tone	$\{(x, y)   x+y=6, 1, 11, 2, 10 \pmod{12}\}$	F2	C4
C4	Non-diatonic succession	Given in the text	F3	C4
C5	Major/minor triad	Given in the text	F4	C2, C5

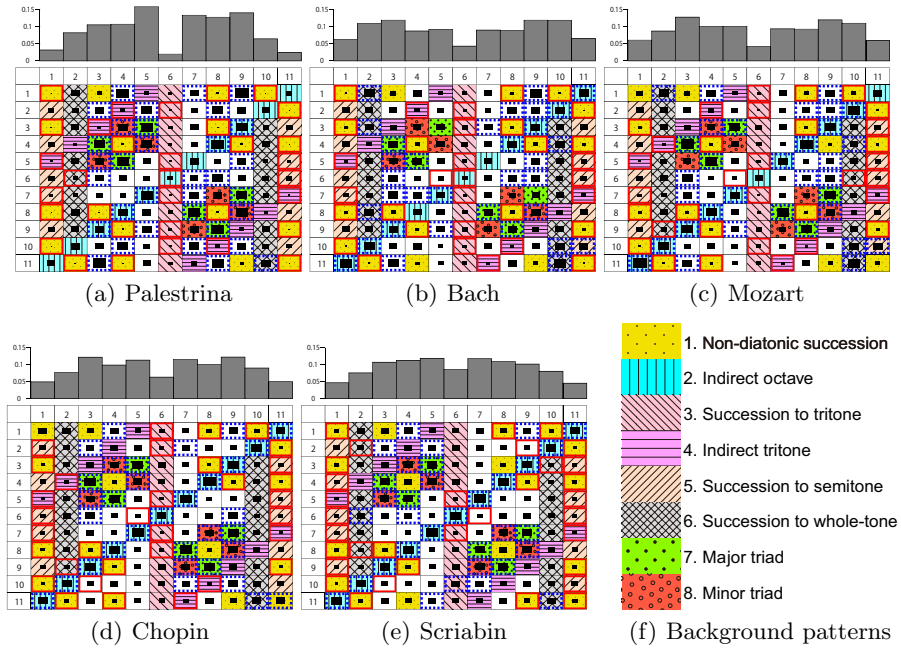
motions. With some calculation, we can find all such “non-diatonic successions”  $(x_n, x_{n+1})$  as (1,1), (1,3), (1,8), (1,10), (2,11), (3,1), (3,8), (4,4), (4,9), (4,11), (8,1), (8,3), (8,8), (9,4), (9,11), (10,1), (11,2), (11,4), (11,9), (11,11). Although it is not always true, a modulation often involves a non-diatonic motion (within a voice or across voices), which induces a non-diatonic PCI succession if it occurs in a small range.

Non-harmonic notes cannot be expressed simply in the sequence of pc intervals without preliminary chord analysis. Nevertheless we can find related PCI successions by paying attention to the opposite notion of “harmonic notes”. The condition of a harmonic note in a strict sense is that the note and the other notes sounding at the same time are contained in a major or minor triad. A major/minor triad can be expressed with a pair of pc intervals. For example, a C major chord can appear as (E,G,C) and its permutations in the sequence of pcs, which is expressed as PCI successions (3,5), (4,3), (5,4), (7,9), (8,7), or (9,8). Similarly a minor triad is expressed as (3,4), (4,5), (5,3), (7,8), (8,9), or (9,7). Another class of PCI successions related to harmonic notes is indirect octave, which is represented as  $(x_n, x_{n+1})$  with  $x_n + x_{n+1} = 0 \pmod{12}$ . Many simultaneous notes in octaves imply that they are harmonic notes. They are related to the number of voices or the chord density, which characterise polyphonic textures.

Table 1 summarises the discussed classes of successions of two pc intervals and their relation to the polyphonic features. The same argument can be applied to successions of three or more pc intervals. Longer successions provide more information, but it is harder to obtain statistically meaningful results when dealing with numerical data. We here concentrate on successions of two pc intervals.

### 3 Markov Model of PC Intervals

The polyphonic features characterise music styles in terms of their frequencies, and statistical models can be used to describe their quantitative nature. A simple statistical model that can describe relations between successions of two data points in a sequence is the first-order Markov model. The (stationary) Markov model of pc intervals is described with an initial probability and transition probabilities, which are given by  $P^{\text{ini}}(x) = P(x_1 = x)$  and  $P(x|y) = P(x_{n+1} = x | x_n =$



**Fig. 1.** Transition probabilities obtained from pieces of Palestrina, Bach, Mozart, Chopin, and Scriabin (a)–(e). Each black square at the centre of the  $(x, y)$ -th cell shows the transition probability  $P(y|x)$  in proportional to the value. For each row in each table, the three highest (resp. lowest) values are indicated with blue dashed (resp. red bold) square frames. The list (f) explains the background patterns and colours of the cells. The distribution of pc intervals  $P(x)$  is also shown above each table.

$y$ ). Due to the ergodicity of a Markov model, the initial probability has little effect for a long sequence, and we here mainly consider the transition probabilities, which have  $11 \cdot 10 = 110$  independent parameters. The distribution of pc intervals  $P(x) = P(x_n = x)$  can be derived from the transition probabilities by the equilibrium equation:  $P(x) = \sum_y P(x|y)P(y)$ . The relative frequencies of a succession  $(x, y)$  is described with  $P(y|x)$ .

Figure 1 illustrates the values of transition probabilities obtained from MIDI data of pieces by five composers, Palestrina, J. S. Bach, Mozart, Chopin, and Scriabin, whose works are usually associated with the period of the Renaissance, the Late Baroque, the Classical, the Early Romantic, and the Late Romantic/Early 20th century. The number of pieces and the data size are shown in Table 2. The background patterns of the cells indicate the classes of the corresponding PCI successions as summarised in Fig. 1(f). When a succession belongs to more than one classes, the upper most class in the list is indicated.

We see that some patterns in the transition probabilities accord with the background patterns of the cells. For example, non-diatonic successions, succes-

**Table 2.** Results of discriminating pieces by five composers with the Markov model with the constrained (resp. full) parametrisation. Each value indicates the rate (%) of pieces recognised as the corresponding composer.

Composer	Data size	Palestrina	Bach	Mozart	Chopin	Scriabin
Palestrina	175 pcs (1.95 MB)	94.9 (99.4)	2.9 (0.6)	1.7 (0)	0.6 (0)	0 (0)
Bach	108 pcs (0.93 MB)	2.8 (0)	77.8 (80.6)	13.0 (13.9)	1.9 (5.6)	4.6 (0)
Mozart	77 pcs (2.40 MB)	2.6 (1.3)	5.2 (3.9)	71.4 (84.4)	15.6 (10.4)	5.2 (0)
Chopin	90 pcs (1.51 MB)	3.3 (1.1)	10.0 (1.1)	15.6 (14.4)	44.4 (67.8)	26.7 (15.6)
Scriabin	102 pcs (0.79 MB)	9.8 (5.9)	15.7 (6.9)	3.9 (2.0)	14.7 (19.6)	55.9 (65.7)

sions to tritone, and indirect tritones generally have small probabilities. Probability values corresponding to these PCI successions are generally larger for composers of later periods, which is a consequence of the time evolution in the use of dissonances. Similarly, other patterns of transition probabilities can be associated with the classes discussed in the previous section, and their tendencies for each composer reflect the quantitative nature of the polyphonic features in different music styles. We omit further details of the analysis for the lack of space.

#### 4 Constrained Parametrisation and Composer Discrimination

To quantitatively examine how much the polyphonic features provide information to characterise different music styles, we compare results of composer discrimination with the Markov model and a reduced model with constrained parameters that are related to the classes of PCI successions. In the constrained model, we introduce five parameters  $p(\text{non-diatonic})$ ,  $p(\text{indirect-octave})$ ,  $p(\text{tritone})$ ,  $p(\text{second})$ , and  $p(\text{triad})$ , which parametrises transition probabilities of class 1, 2,  $\{3, 4\}$ ,  $\{5, 6\}$ , and  $\{7, 8\}$  in Fig. 1(f). The rest probabilities  $P(x|y)$  are assumed to be uniform for each  $y$  and determined by the normalisation of probabilities  $\sum_x P(x|y) = 1$ . An algorithm to discriminate composers can be developed from these models with the maximum likelihood estimation.

Results of composer discrimination are shown in Table 2. To avoid statistical artefacts by overfitting, the piece-wise leave-one-out method was used. The composer-wise averaged accuracy was 68.9% (resp. 79.6%), and the mean reciprocal rank (MRR), which is the averaged reciprocal rank of the correct composer, was 1/1.20 (resp. 1/1.11) for the constrained (resp. full) parametrisation. Compared to the reduction of parameters (110 to five), there was a small decrease of the accuracy for Palestrina and Bach, and a rather large (but not very large) decrease for the other composers. For Chopin and Scriabin, we see that a large proportion of misclassified pieces are associated with adjacent composers in the table for both models.

The rather high accuracies of the full Markov model indicate that the model parameters well capture characteristics of the composers, and the not-heavy de-

terioration of accuracies with the reduced model indicates that a significant part of the characteristics is associated with the polyphonic features. The overall tendency that misclassified pieces were more frequently classified to a composer that is near in the lived period implies that the features capture not only particular styles of the composers but also a generic style of the composed period to some extent, which confirms the general intuition about the evolution of music styles.

## 5 Discussion

It is interesting to apply the present analysis for music style classification problems. The current state-of-the-art classification algorithms naturally employ many features related to pitch and rhythm [8–10], and the use of the polyphonic features and the pcs intervals would improve the accuracy, computational efficiency, and generality. It would be possible to construct an effective classification algorithm applicable for general polyphonic MIDI data including performance recordings, for which information on voice and rhythm cannot be extracted directly. To our knowledge such an algorithm has not been proposed so far.

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## References

1. Jeppesen, K.: The style of Palestrina and the dissonance. (2nd ed.) Dover Pub., New York (2005) (Originally published by Oxford Univ. Press in 1946.)
2. Kostka, S., Payne, D., Almén, B.: Tonal harmony (7th ed.). McGraw-Hill, New York (2004)
3. Tymoczko, D.: A geometry of music. Oxford University Press, New York (2011)
4. Hu, D.: Probabilistic topic models for automatic harmonic analysis of music. Ph. D. Assertion, UC SanDiego (2012)
5. Handelman, E., Sigler, A.: Key induction and key mapping using pitch-class set assertions. In Proc. MCM, pp. 115–127. Springer, Berlin Heidelberg (2013)
6. Honingh, A., Bod, R.: Clustering and classification of music using interval categories. In Proc. MCM, pp. 346–349. Springer, Berlin Heidelberg (2011)
7. Wolkowicz, J., Kulka, Z.: N-gram based approach to composer recognition. Archives of Acoustics 33.1, pp. 43–55 (2008)
8. Hillewaere, R., Manderick, B., Conklin, D.: String quartet classification with monophonic models. In Proc. ISMIR, pp. 537–542. (2010)
9. Hasegawa, T., Nishimoto, T., Ono, N., Sagayama, S.: Proposal of musical features for composer-characteristics recognition and their feasibility evaluation (in Japanese). J. Information Processing Soc. of Japan Vol. 53, No. 3, 1204–1215 (2012)
10. Conklin, C.: Multiple viewpoint systems for music classification. Journal of New Music Research, Vol. 42(1), 19–26 (2013)
11. MIREX (Music Information Retrieval Evaluation eXchange) homepage: [http://www.music-ir.org/mirex/wiki/MIREX\\_HOME](http://www.music-ir.org/mirex/wiki/MIREX_HOME)